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Automatically Extraction and Reconstruction of Cupola Geometries of Orthodox Churches from Precision Point Clouds

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Abstract: Complex geometry extraction from point clouds is an actual problem in reverse engineering. Simple geometrical models (like parallelepipeds, prisms, pyramids, cones, spheres) were already applied in construction and machine-building modeling, but are not sufficient for high quality BIM now. This work, which is carried out in the context of virtual reconstruction of destroyed orthodox churches, presents a robust and efficient method of cupola (domes) and tambour geometry extraction from precise point clouds. The rich diversity of architectural forms, which are defined by many parameters, does not allow to consider this problem as a trivial duty, because usual geometry extraction methods fail for these object types. The new developed algorithm is presented and realized.

1 Introduction

1.1 Motivation

Ancient cultural and science developments, accumulations of practical human experience, led to fantastic achievements in industrial and civil constructions. These achievements are reflected in various architectural forms, construction materials, and technologies.

Simple geometrical models (like parallelepipeds, prisms, pyramids, cones, spheres) were already applied in construction and machine-building modeling. However, such geometrical forms are insufficient for modern industry requirements, which arises the need for more difficult mathematical models, and, therefore, the identification relations between the parameters of the geometrical models and the projected constructions.

This work is carried out in the context of actual research in the virtual reconstruction of destroyed orthodox churches, in which the cupola (dome) geometry extraction plays an important role. The rich diversity of architectural forms does not allow us to consider this problem as a trivial duty.

Analytical surfaces have a broad application in the various branches of technique and construction, being relevant for the geometry description of complex church components. A significantly high

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number of parameters, which complicate a geometry detection task from a point cloud, defines such kind of surfaces.
The aim of this work is the development of a robust method of automatically dome geometry extraction from precision point clouds.

1.2 Related works

Complex geometry extraction from point clouds is a current problem in reverse engineering, that is considered in many articles. Due to the huge number of parameters in the context of big data problems, which is relevant for point clouds, the usual geometry extraction methods (e.g. extraction of geometrical primitives with Hough Transform or RANSAC) fail in this case. Moreover, our research object cannot be approximated with geometrical primitives in most cases (unlike to e.g. ALBY & GRUSSENMEYER 2012).
The mathematical construction of complex surfaces is one of the main objectives in engineering geometry. Different mathematical models and aspects of curve and complex surface approximation from point clouds are represented in BUREICK et al. (2016).
A spline interpolation is very common in point cloud processing (e.g. thin plate spline in non-rigid texture mapping procedure (FAN et al. 2012)) and curve fitting (WANG et al. 2004). BARAZZETTI et al. (2016) presents a modelling of bridges by NURBS surfaces. B-spline curves are extracted from an input point cloud, using its point subsets as control points for a curve approximation. B-spline basis functions are defined on the knot vector, which is a non-decreasing sequence of real numbers with external elements, the knots. BELYAEVA (2014) deals in her PhD-Thesis with a development of mathematical surface models (vector/matrix models) and surface transformation algorithms for practical tasks with an application of computer geometry. In particularly, the construction of domes in orthogonal and cylinder coordinate systems and its approximation with cubic spline is considered.
However, an important modeling objective in the context of geometry detection from big point clouds is the determination of the minimal number of parameters, which describe necessary variations of dome geometries. Therefore, the extraction of significant information (e.g. features points) is a crucial task.
RUSU et al. (2008) developed a point descriptor, identifying points on planar, round, linear surfaces. In BUENO et al. (2016) the key points are detected using entropy values by planarity and curvature change. In GEVAERT et al. 2016 a maximal height difference, its standard deviation and number of points per bin are estimated, solving for key point extraction of an input point cloud. The bins have been defined from the geographical grid determined by a orthomosaic. Some algorithm, which are suitable for feature point detection in 2D images (Harris detector (HARRIS & STEPHENS 1988), SIFT (LOWE 1999; LOWE 2004)) are applicable for 3D point clouds, too.
There are some program applications allowing a semi-automatic modeling of architectural forms, in which the model parameters of the structure elements are estimated from user defined keypoints (KIVILCIM & DURAN 2016). Such approaches basis on an a priori created library of architectural elements (DORE et al. 2015). DORE et al. (2015) estimated and reconstructed destroyed architectural elements (domes and tambours) from horizontal cuts, according to a model from elements library.

CANCIANI et al. (2015) used point cloud cuts to extract key points and estimate a mathematical model of architectural elements for its further "path-wise" reconstruction.
According to our objectives, we want to provide a complex form reconstruction algorithm using a minimal parameter set, that will be suitable for incomplete data, too.

2 Method

The proposed method can be divided in two steps – dome block segmentation and dome/tambour geometry extraction. The second step is in the focus of this article and will be described according to the research object.

2.1 Pre-segmentation of dome blocks

The algorithm, considered in CHIZHOVA ET AL. 2016, bases on the idea, that there is a certain number of domes of orthodox churches (e.g. 1, 3, 5, 7, 9, 33). The quantity of domes is determined by orthodox construction canons. Our duty is to segment the point set $P_{x,y,z}$ into subsets $P_{i,}$, which may belong to every type but also to a single dome. The number of such subsets corresponds to the possible number of the domes.
We consider horizontal cuts of the treated church point cloud, which are covered with an orthogonal grid. In each layer, at different heights, containing the point cloud cut of the dome and the grid, distances between grid nodes and cut points are estimated. If some points have almost an equal distance to the grid nodes, these points are assumed as a dome/tambour with a circular horizontal projection. A user defined deviation of the distances gives an "index of detection quality", which maximal value allows to state, that the bottom of the dome block has been detected.

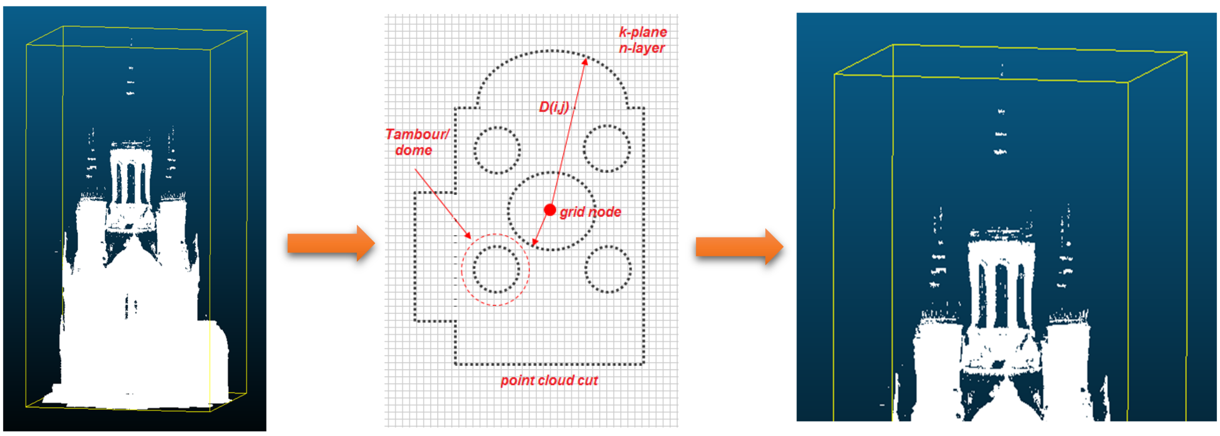

Fig. 1: Segmentation process of dome blocks (dome with tambour)

2.2 Mathematical modelling and point cloud simulation of domes

In this research, the mathematical model of the dome should consider the specific properties of point cloud analysis as well as the classification and storage procedure of recognized objects. Therefore, it is necessary to reduce a space dimension, in which an analysis is carried out, and to minimize a set of model parameters, keeping its generality. As a basic form of a dome, we will choose the most common onion form, which can be considered more universal in relation to other domes forms (e.g. oval domes). Besides, it is necessary to consider such types of domes (and its tambours) like ripped or "umbrella" domes, which are divided at the base into curved segments, which follow the curve of the elevation.

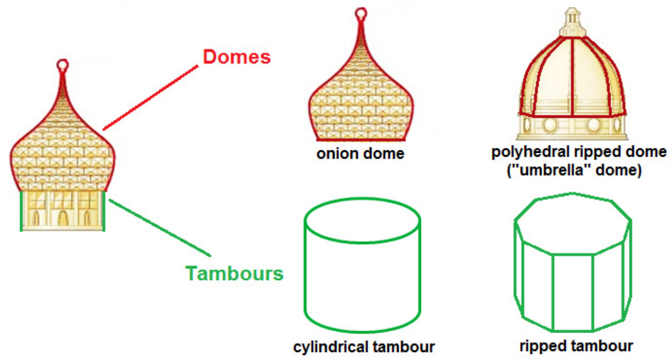

Fig. 2: Different types of domes and tambours

Obviously, the onion domes have an axial symmetry, being a surface of rotation, which are defined completely by a planar profile curve (meridian).

Therefore, the modeling of a dome form can be considered as a selection of a planar parametric curve, setting the meridian. Let's direct the abscissa axis along the dome axis from its basis to its top, then a radius of the dome cuts on different heights will be stored on the ordinate axis. Moving along the dome axis, the radius of its cut increases at first, reaching a maximum, then decreases, concerning an axis on its top. Obviously, there is a crease point between a maximum and top.

This kind of geometrical forms is described usually with parametric third order Bezier curves or cubic splines. However, in our case, this representation form cannot be used, because a parameter set of Bezier curve includes not only the points on this curve but external point too, setting its tangent and providing only directional information (a curve does not pass through these points).

Obviously, the reconstruction procedure of a tangent to the dome surface in the point cloud will have an error, which is bigger as an error by the definition of point coordinates on the dome surface. But a specific feature of the Bezier curves is, that the allocation of parametric tangents has more influence on a curve form as the coordinates of a points on this curve.

Moreover, a standard set of tangent segments with coordinates of its end points cannot be used in our case, because it will complicate a procedure of object classification and storage. A tangent setting with normalized vectors can solve as makeshift, but it will not improve a situation with error determination.

An application of a cubic spline is problematic in this case, too. It is known, that cubic spline parameters significantly depend on a piecewise-defined interval or its knots, building this spline. An effective classification will be interfered through a problem to offer a spline composition, uniform for all reconstructed dome forms.

Obviously, all listed above problems are insoluble in case of a geometry reconstruction from incomplete data, if a dome is partially destroyed or scanned with a big error.

Assuming enough simple prerequisites, it is possible to approximate a dome profile curve with a third degree polynomial, which coefficients are determined by coordinates of four reference points. In this case, the coefficient values or polynomial roots will be invariant concerning the choice of these reference points that allows using of polynomial coefficients or its roots as form parameters by object classification and storage.

Following points are chosen as reference (features) points:

- Dome basis: a point on the junction from the tambour to the dome ($\boldsymbol{P_0(x_0,y_0)}$);
- Equator: a point on the maximal radius of the profile curve ($\boldsymbol{P_1(x_1,y_1)}$);
- The crease point by the function change of the profile curve ($\boldsymbol{P_2(x_2,y_2)}$);
- The point on the dome top ($\boldsymbol{P_3(x_3,y_3 = 0)}$)

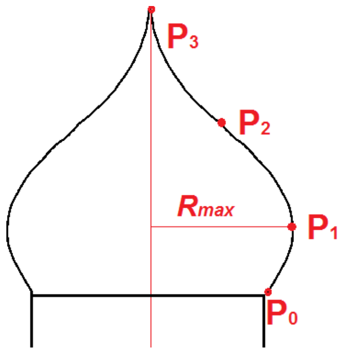

Fig. 3: Reference points on dome profile to be detected

Obviously, a point $\boldsymbol{P_3}$ corresponds to one of the polynomial roots; therefore, a dome top can be chosen as one of the polynomial parameters and coefficients ***a, b, c*** of the second order polynomial as residual parameters. In this case, a profile curve model can be defined as

$$f(x)=(ax^2+bx+c)(x-x_3)$$

This model was investigated during extensive computing experiments, in which course it has become clear that the cubic polynomial is not suitable due to wrong approximation of $\boldsymbol{P_3}$ point vicinity. In particular, a condition of the profile contact with dome axis is violated, but it cannot be permitted by the construction rules of domes. Thus, a polynomial order has been raised to the fourth without increasing the parameter number for faults elimination. It is possible, if the difference $\boldsymbol{(x-x_3)}$ will be squared.

Thus, the model can be defined by multiplication of second order polynomial to squared difference:

$$f(x)=(ax^2+bx+c)(x-x_3)^2$$

Due to x_3-root multiplicity, the total number of parameters will remain equal to four; it is necessary to determine three residual unknown model parameters ***a, b, c*** (x_3 is already known). All calculation formulas have been received using a computer algebra system and exported into the C++ programming language. It allows to construct a generator for the simulation of domes point clouds. The program realization of the test data generator and geometry analyzer is made in a cross-platform integrated environment of software development (IDE) Qt5 using the PCL library version 1.8.

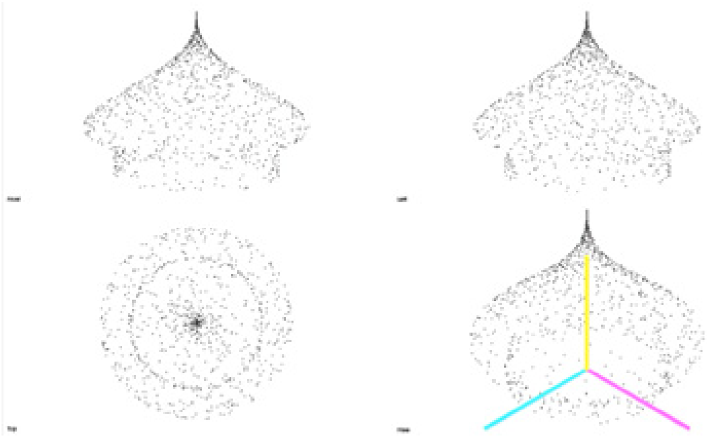

Fig. 4: Generated point clouds of domes

2.3 Extraction and reconstruction of the dome geometry

A detection procedure of dome the geometry consists of the following steps:

- Point cloud regularization;
- Definition and refining of dome axis coordinates;
- Profile curve extraction;
- Definition and refining of model parameters.

2.3.1 Point cloud regularization

Point cloud regularization allows reducing a point cloud capacity to values according to given spatial resolution and – therefore – modeling accuracy that relieves excessive calculations and increases the program efficiency. Moreover, it allows to eliminate the potential irregularity of the input point cloud. This procedure splits the point cloud space with cubic voxel grids. Further, all points in each voxel have been replaced with their centroides, which coordinates are defined by the average of such point coordinates.

However, in case of incomplete input data, it can be insufficiently for the construction of the profile curve. In particular, this procedure does not guarantee that the centroid of the considered layer will coincide surely with the center of the circle, being a dome cut in this layer.

2.3.2 Dome axis extraction

On the next step, a layered bottom-up "scanning" of the grid will be carried out for the dome axis definition and creation of its array. All horizontal grid layer, which include at least three points, will be analyzed with the following algorithm:

1) At first, a projection of all layer points to the plane $z=0$ will be calculated;
2) Then the initial proximity of the center and the circumcircle of these points will be defined. Three points will be chosen for this purpose according to the following rule: at first we choose two points in the point set, lying on the maximal distance from each other, then we find a third point, which is maximal distant from each of the first two points. Based on this data, the center coordinates and radius of the circle, set by the chosen points, are calculated. Such procedure allows to increase the definition accuracy of the initial parameter proximity in case of incomplete input data.
3) Further, circle parameters are specified by the conjugate gradient method for the minimization of the sum of squared distances from the circle to all points in this layer. The calculated coordinates of the circle center are accepted as dome axis coordinates in this layer and saved for a subsequent averaging on all analyzed layers.
4) Then we search for maximum and minimum distances from this center to the layer points. It is necessary for detecting of possible polygonal forms in the dome cut.
   A maximal distance is accepted as radius of regular polygon's circumcircle and saved in the array element of profile curve according to an analyzed layer.
   A minimal distance is considered as the incircle radius of the same polygon using for the assessment of polygon sides number (n) with the following formula:

$$n=180^0/\arccos(R_{min}/R_{max})$$

Besides, an angle, which set a direction on the first found maximum, will be estimated in polar coordinate system.

Further, these parameters are specified by conjugate gradient method for deviation's minimization of all layer points from constructed polygon. The calculated values will be stored for the subsequent averaging on the analyzed layers, which is carried out separately for dome and tambour.

5) After the "scanning" of the layers, which are important for the analysis, we build up an array containing information about layer height, maximal cut radius in the layer and polygon parameters. Average coordinate values of cut centers are accepted as dome axis allocation.

Then, an array analysis will be carried out for the parameter assessment of profile curve.

2.3.3 Profile extraction and model definition

A profile curve and its first derivative, defined by numerical differentiation, are needed for steady searching of reference (feature) points. A second derivative of profile curve could be useful for point $\boldsymbol{P_2}$ search, but preliminary experiment shows, that an error of its values, defined by a numerical method, is very big due to noise with accidental outliers.

Therefore, only the function and its first derivative are analyzed. Array recognition is carried out according to increasing layer height.

The following feature points are detected from function and derivative analysis:

1) the point $\boldsymbol{P_0}$ is detected by the maximal value of first derivative;
2) the point $\boldsymbol{P_1}$ is detected by the maximal value of the function;
3) the point $\boldsymbol{P_2}$ is detected by the minimal value of first derivative and
4) the point $\boldsymbol{P_3}$ is detected by the maximal height value of dome point cloud.

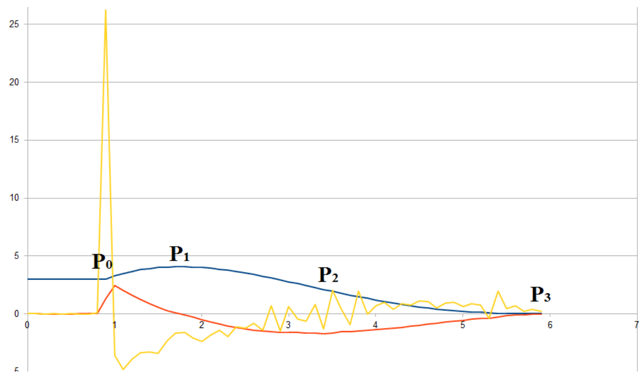

Fig. 5: Derivative analysis of extracted profile curve

The model parameters $\boldsymbol{a, b, c, x_3}$ are calculated from the coordinates of found points and specified by a least squares method. Thus, a profile curve will be constructed based on this data.

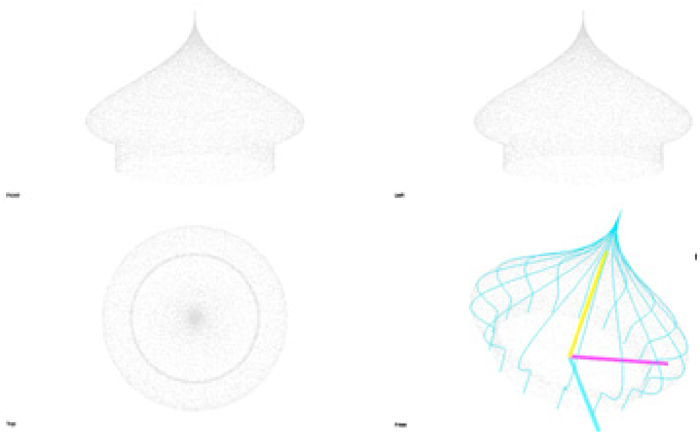

Fig. 6: Reconstructed profile curve from point cloud

In the final step, the averaging of polygon parameters is carried out for the dome and the tambour. A cut is polygonal, if the number of sides $2 < n < 14$. In all other cases it will be considered as a circle.

For the simplification of the classification problem, it could be better to store profile curve parameters instead of the coordinates of reference (feature) points. For this purpose, it is necessary to normalize coordinates of these points on the dome height, which is defined by the optimization, to repeatedly calculate and store profile parameters based on this normalization.

3 Conclusion

This article presents a robust and efficient method of the automatic extraction and reconstruction of complex geometrical cupola forms from point clouds. An example has been shown.

Practically, a calculation of the second derivation is not necessary because of oscillations of discrete point cloud. The algorithm shows a good geometry reconstruction in practical application.

4 References

ALBY, E., & GRUSSENMEYER, P., 2012: From point cloud to 3d model, modelling methods based on architectural knowledge applied to fortress of chatel-sur-moselle (france). – International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences **39**(B5), 75-80.

BARAZETTI, L., BANFI, F., BRUMANA, R., PREVITALI, M. & RONCORONI, F., 2016: Bim from laser scans... not just for buildings: Nurbs-based parametric modeling of a medieval bridge. ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences **III**(5), 51-56.

BELYAEVA, Z., 2014: Geometric modelirovanie prostranstvennyh konstrukziy. PhD thesis. Uralische Föderale Universität benannt nach dem ersten Präsidenten Russlands B.N. Jelzin, Jekaterinburg, Russland.

BUENO, M., MARTINEZ-SANCHEZ, J., GONZALEZ-JORGE, H. & LORENZO, H., 2016: Detection of geometric keypoints and its application to point cloud coarse registration. The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences **41**(B3), 187-194.

BUREICK, J., NEUNER, H., HARMENING, C. & NEUMANN, I., 2016: Curve and surface approximation of 3d point clouds. Allgemeine vermessungs-nachrichten (avn) **123**(11-12), 315-327.

CANCIANI, M., FALCOLINI, C., SACCONE, M. & SPADAFORA, G., 2013: From point clouds to architectural models: algorithms for shape reconstruction. The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences **40**(5/W1), 27-34.

CHIZHOVA, M., KOROVIN, D., BRUNN, A. & STILLA, U., 2016: Automatical segmentation of cupola blocks from unorganized point clouds. Proccedings of GeoSpace Congress in Kiev.

DORE, C., MURPHY, M., MCCARTHY, S., BRECHIN, F., CASIDY, C. & DIRIX, E., 2015: Historic Building Information Model (HBIM). The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences **40**(5/W4), 351-357.

FAN, Z., XIANFENG, H., ZHICHAO, Z., WEI, F. & DEREN, L., 2012: High precision texture reconstruction for 3d sculpture model. International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences **39**(B5), 139-143.

GEVAERT, C., PERSELLO, C., SLIUZAS, R. & VOSSELMANN, G., 2016: Classification of informal settlements through the integration of 2d and 3d features extracted from UAV data. ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences **III**(3), 317-324.

HARRIS, C. & STEPHENES, M., 1988: A combined corner and edge detector. Proceedings of Fourth Alvey Vision Conference, 147-151.

KIVILCIM, C. & DURAN, Z., 2016: A semi-automated point cloud processing methodology for 3d cultural heritage documentation. The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences **41**(B5), 293-296.

LOWE, D., 1999: Object recognition from local scale-invariant features. Proceedings of the Seventh IEEE International Conference on Computer Vision **2**, 1150-1157.

LOWE, D., 2004: Distinctive image features from scale invariant keypoints. International Journal of Computer Vision **60**(2), 91-110.

QUATTRINI, R., MALINVERNI, E. S., CLINI, P., NESPECA, R. & ORLIETTI, E., 2015: From TLS to BIM. High Quality semantically-aware 3D Modelling of Complex Architecture. The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences **40**(5/W4), 367-374.

RUSU, R., BLODOW, N., MARTON, Z. & BEETZ, M., 2008: Aligning point cloud views using persistent feature histograms. Proceedings of IEEE/RSJ International conference in Intelligent Robots and Systems, 3384-3391.

WANG, W., POTTMANN, H. & LIU, Y., 2004: Fitting b-spline curves to point clouds by squared distance minimization. Technical report. HKU CS Tech Report TR-2004- 11, http://www.cs.hku.hk/research/techreps/document/TR-2004-11.pdf.